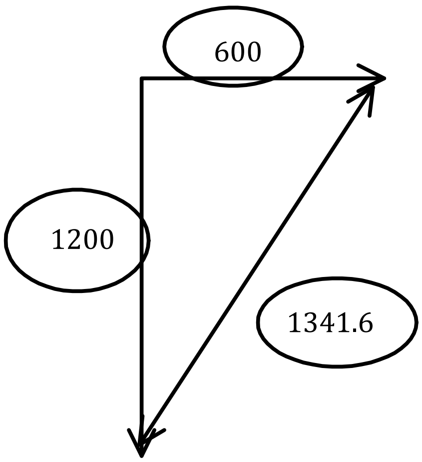
C12 - 4.2 - Train Pythag/Spotlight Sim Tri Rel Rat Notes

Train 'a' leaves Vancouver heading South at 10 m/s and train 'b' leaves heading East at 5 m/s? How far are they a part after two minutes? What is the speed at which the trains are moving apart at that time?



$$\frac{da}{dt} = 10 \qquad \frac{db}{dt} = 5 \qquad \frac{dc}{dt}\Big|_{t=2} = ?$$

$$a^2 + b^2 = c^2$$
$$1200^2 + 600^2 = c^2$$
$$c = 1341.6$$

$$a^2 + b^2 = c^2$$
$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$
$$2(1200)(10) + 2(600)(5) = 2(1341.6)\frac{dc}{dt}$$
$$30000 = 2683.2\frac{dc}{dt}$$
$$\frac{dc}{dt} = 11.1\frac{m}{s}$$

$2\ minutes = 120\ seconds$

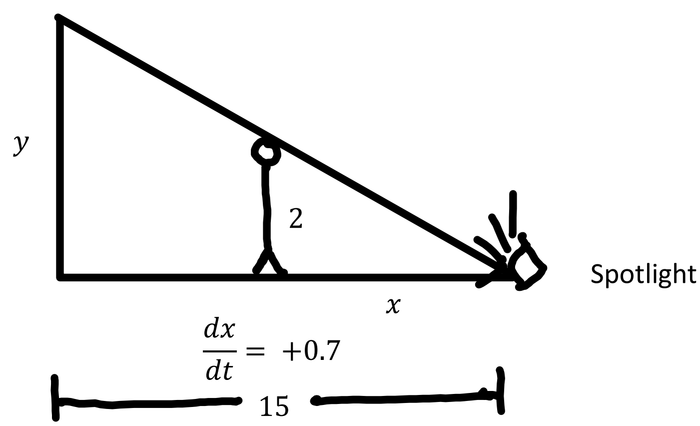
$$a = vt$$
$$a = 10 \times 120$$
$$a = 1200$$

$$b = vt$$
$$b = 5 \times 120$$
$$b = 600$$

$$d = vt$$

A 2 m tall person is walking away from a spotlight, 15 m from a wall, towards the wall at 0.7 m/s. How fast is the shadow on the wall changing when they are 7 m from the spotlight?

$$\frac{dy}{dt}\Big|_{x=7} = ?$$



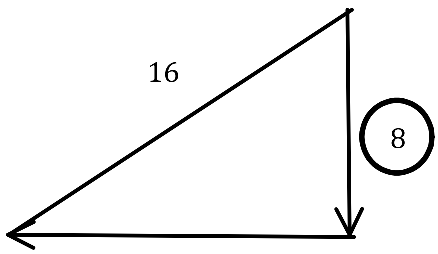
$$\frac{dx}{dt} = +0.7$$

$$\frac{y}{15} = \frac{2}{x}$$
$$xy = 30$$
$$\frac{dx}{dt}y + \frac{dy}{dt}x = 0$$
$$0.7(4.29) + \frac{dy}{dt}(7) = 0$$
$$\frac{dy}{dt} = -\frac{0.7(4.29)}{7}$$
$$\frac{dy}{dt} = -0.429\frac{m}{s}$$

$$xy = 30$$
$$7y = 30$$
$$y = \frac{30}{7}$$
$$y = 4.29$$

C12 - 4.2 - Ladder Trig Related Rates Notes

The top of a 16 ft ladder slides down a wall at a rate of 3 ft/s. At what rate is the base of the ladder sliding away from the wall when the latter is at a height of 8 ft on the wall.



$$\frac{dy}{dt} = -3\frac{ft}{s}$$

*Length is shrinking: Derivative is Negative.

$$\frac{dx}{dt}\Big|_{y=8} = ?$$

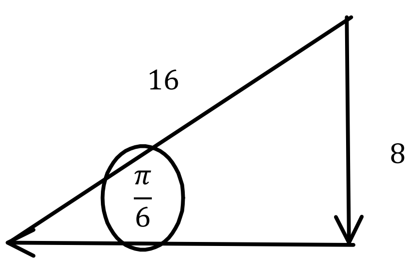
$$x^2 + y^2 = c^2$$
$$x^2 + 8^2 = 16^2$$
$$x = \sqrt{16^2 - 8^2}$$
$$x = \sqrt{192}$$
$$x = 8\sqrt{3}$$

$$x^2 + y^2 = c^2$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2c\frac{dc}{dt}$$

*We can substitute constants into the formula

$$2(8\sqrt{3})\frac{dx}{dt} + 2(8)(-3) = 0$$
$$\frac{dx}{dt} = \frac{3}{\sqrt{3}}$$
$$\frac{dx}{dt} = \sqrt{3}\frac{ft}{s}$$

What is the rate the angle at the bottom of the ladder changing?



$$cos\theta = \frac{x}{r}$$
$$cos\theta = \frac{x}{16}$$
$$-sin\theta\frac{d\theta}{dt} = \frac{1}{16}\frac{dx}{dt}$$
$$-\frac{8}{16}\frac{d\theta}{dt} = \frac{1}{16}\sqrt{3}$$
$$\frac{d\theta}{dt} = -\frac{\sqrt{3}}{8}\frac{rad}{s}$$

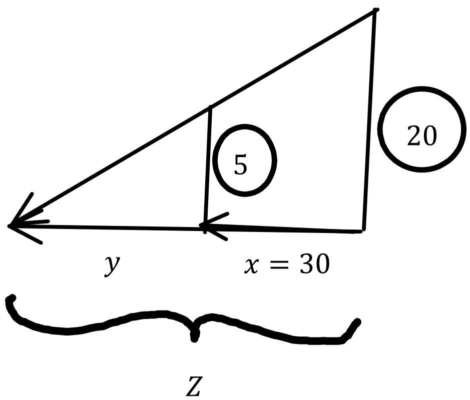
*I used cos because it used the rate I already solved on the top. Using sin and tan is possible but much more difficult based on the information and previously solved. We want our constant on the bottom.

$$sin\theta = \frac{8}{16}$$
$$\theta = \sin^{-1}(\frac{1}{2})$$
$$\theta = \frac{\pi}{6}$$

*Real life is in Radians. Degrees are for children.

C12 - 4.2 - Similar Triangles/Cos Law Related Rates Notes

A 5 foot tall woman is walking away from a 20 foot lamp post at 3 m/s. What rate is her shadow increasing when she is 30 feet from the lamp post; and is her shadow getting bigger or smaller. How fast is the tip of her shadow moving?

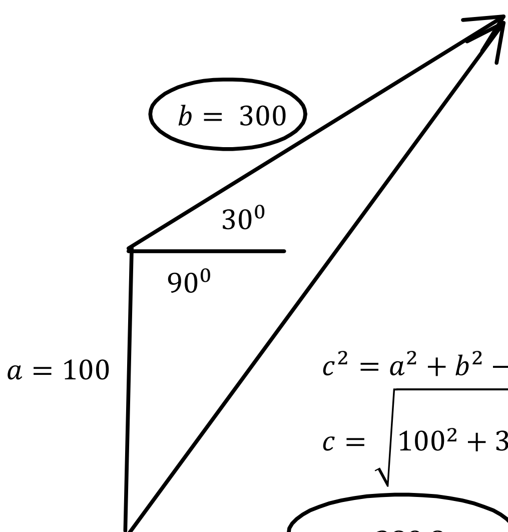


$$\frac{dx}{dt} = 3\frac{m}{s} \qquad \left.\frac{dy}{dt}\right|_{x=30} = ?$$

$$\frac{5}{20} = \frac{y}{x+y}$$
$$5x + 5y = 20y$$
$$5x = 15y$$
$$x = 3y$$
$$\frac{dx}{dt} = 3\frac{dy}{dt}$$
$$3 = 3\frac{dy}{dt}$$
$$\frac{dy}{dt} = 1\frac{ft}{s}$$

$$\frac{dz}{dt} = \frac{dy}{dt} + \frac{dx}{dt}$$
$$\frac{dz}{dt} = 1 + 3$$
$$\frac{dz}{dt} = 4\frac{m}{s}$$

A float plane rising at 30 degrees above the horizontal flies over a boat at an altitude of 100 m at 60 m/s. How fast is the distance between the boat and the plane increasing after five seconds?



$$\frac{db}{dt} = 60 \qquad \frac{dc}{dt}\Big|_{t=5} = ?$$

$$v = \frac{d}{t}$$
$$d = vt$$
$$d = 60 \times 5$$
$$d = 300\ m$$

$$c^2 = a^2 + b^2 - 2abcosC$$
$$c = \sqrt{100^2 + 300^2 - 2(100)(300)\cos\frac{7\pi}{6}}$$
$$c = 389.8\ m$$

*Word Problems in Radians $\quad 120^0 = \frac{7\pi}{6}$

*That would have been a tough product rule if more things were changing

$$c^2 = a^2 + b^2 - 2abcosC$$
$$2c\frac{dc}{dt} = 0 + 2b\frac{db}{dt} - 2acosC\frac{db}{dt}$$
$$2(389.8)\frac{dc}{dt} = 0 + 2(300)(60) - 2(100)\left(-\frac{\sqrt{3}}{2}\right)(60)$$
$$\frac{dc}{dt} = 59.5\frac{m}{s}$$